Particle Physics at the Frontier Lecture 25

In some sense, with the discovery of the Higgs boson, the Standard Model, and therefore particle physics, is complete. This view is more popular than you might think, but is very unfortunate. Just because we have the Standard Model, doesn't mean that we understand the origin of mass, the matter-anti-matter asymmetry, the emergent phenomena of the strong force, how and/or why electroweak symmetry was broken, the mathematical structure of quantum field theories, why and what the dark matter is, or innumerable other problems, including what lies beyond. In this, the final lecture, I will discuss some of the problems yet to solve, which I hope you can contribute to!

Neutrino Masses

We discussed the issue of neutrino masses last week. The fact that different flavors of neutrinos oscillate into one another means that some of the neutrinos must have non-zero masses. This is not allowed by the structure of the Standard Model, which requires neutrinos to be exclusively left-handed fermions. With only left-handed neutrinos in the game, it is not possible to include a mass for them in the Standard Model.

This isn't to say that neutrino masses invalidate the Standard Model; at typical energies neutrinos can be thought of as massless, to extremely good approximation. Adding a mass for neutrinos in the standard Model is actually not that hard, either. Most naively, we can just add a neutrino mass term to the Standard Model. The weak interactions still only couple to left-handed neutrinos, and this is the only way that we know how to create neutrinos.

That is, in any reaction that we study, left-handed neutrinos will be produced, that then very slowly mix into right-handed neutrinos as they propagate. This is not inconsistent with any observations because in neutrino oscillation experiments, we are not sensitive to the spin of the neutrinos.

If we included such a term in the Standard Model, then the neutrino and anti-neutrino are distinct particles. Such a mass term is called a "Dirac mass". However, it is perfectly consistent with all symmetries to postulate that neutrinos are their own anti-particle. Neutrinos are neutral, and so their anti-particle is also neutral. (This is the reason why the electron cannot be its own anti-particle: the electron and positron have different charge.) In the Standard Model, an example of a particle that is its own anti-particle is the photon or Z-boson. One can write down a mass for the neutrino as its own anti-particle; this mass term is called a "Majorana mass" after Ettore Majorana. Majorana disappeared under exceptionally suspicious circumstances traveling by boat from Palermo to Naples in 1938. Though only 32 when he disappeared, Fermi considered him a peer of Newton and Einstein.

There are experiments that are currently running that are looking for evidence of neutrinos that are their own anti-particle, called "Majorana neutrinos". The largest experiment, called EXO (Enriched Xenon Observatory) consists of an enormous vat of enriched ^{136}Xe. The vat is observed continuously for evidence of a double β-decay of xenon to barium:

$$^{136}Xe \to {}^{136}Ba + e^+ + e^- + \nu_e + \bar{\nu}_e$$

This decay has been observed and has a lifetime of about 10^{21} years. If the neutrino is its own anti-particle then it is possible for there to be no neutrinos produced in the final state:

$$^{136}Xe \to {}^{136}Ba + e^+ + e^-$$

This decay is called "neutrinoless double β-decay". The distinction between neutrinoless double β-decay and standard β-decay is that the emitted electron and positron should have equal and opposite momentum, as there are no final state neutrinos. EXO is looking for this but hasn't found anything yet. Stay tuned...

Higgs Self-Coupling

In our understanding of the Standard Model, we set up the potential of the Higgs boson so that the origin, when $\varphi = 0$, is an unstable equilibrium point. The Higgs then "rolls" down the potential and settles in the minimum. Because $\langle \varphi \rangle \neq 0$, this spontaneously breaks the electroweak gauge symmetry, giving the W and Z bosons a mass. We argued that this unified electroweak theory had a lot of constraints. What was not constrained, however, was the precise shape of the Higgs potential. Can we nail down this mysterious piece of the Standard Model?

Let's remind ourselves what the scalar potential of the Higgs boson was in the Standard Model. The potential is:

$V(\vec{\varphi}) = \lambda\left(|\vec{\varphi}|^2 - \frac{v^2}{2}\right)^2$, where λ is the quartic coupling of the Higgs and v is the vev. Expanding about v as

$\vec{\varphi} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} + \frac{h(x)}{\sqrt{2}} \end{pmatrix}$, the spontaneously-broken potential

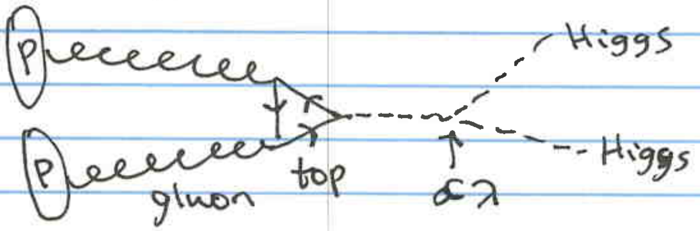
is: $V(h) = \frac{\lambda}{4}\left(h^2 + 2vh\right)^2 = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$.

The mass of the Higgs boson is $m_H^2 = 2\lambda v^2$ and so the quartic coupling is

$$\lambda = \frac{m_H^2}{2v^2} = \frac{(125\,\text{GeV})^2}{2(246\,\text{GeV})^2} = 0.13.$$

I've used the mass of the Higgs boson of $m_H = 125$ GeV and the value of the vev at $v = 246$ GeV.

Currently as discussed in the previous lecture, all we know about the Higgs boson is its mass, its spin, and (some of) its decay modes. This is not enough information to determine the shape of the potential, and therefore the mechanism for the breaking of electroweak symmetry. To determine the Higgs potential requires measuring self-interactions of the Higgs boson. For example, the production of two Higgs bosons in the final state will have a contribution from a diagram that is proportional to λ directly:



Similarly, while I won't draw it, the production of three Higgs bosons in the final state has a contribution that is also sensitive to λ directly. Therefore, the measurement of the cross-section for multi-Higgs production is sensitive to the <u>shape</u> of the potential.

Multi-Higgs production has not yet been observed at the LHC, and it is likely that it will only be measured at a future experiment. Nevertheless, this will be a hugely exciting test for the Standard Model. Additionally, when and/or if observed, it will be the first direct measurement of a fundamental particle's* self-interaction!

Another interesting question is how the potential got to be formed in the first place. The standard picture is that in the early universe, when it was very hot, the vev of the Higgs was 0: $\langle \vec{\varphi} \rangle = 0$. As the temperature cooled, there was a phase transition (like water into ice) that modified the potential and gave the Higgs a non-zero vev: $\langle |\vec{\varphi}| \rangle = v$. The exact dynamics of this phase transition depends on properties of the early universe. So, if we are able to measure Higgs self-interactions, this could provide information about what was happening right after the Big Bang! Cool!

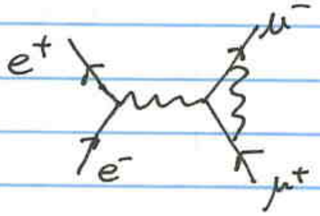
End of Feynman Diagrams

Throughout this class, we have used Feynman diagrams as the language in which we expressed processes in particle physics. Feynman diagrams are our physical picture, they have a precise mathematical formulation, and they are systematically improveable as a perturbation theory.

Behind this beautiful façade lies an ugly truth: the perturbation theory of Feynman diagrams does not converge. There are other ugly aspects which we will come to in a second.

The claim that Feynman diagram perturbation theory doesn't converge is a big one, but easy to prove. Feynman diagrams are ordered by the number loops they have. Each loop in a Feynman diagram adds another factor of the coupling constant, like α in QED or α_s in QCD. To see this compare the tree-level to a one-loop diagram for $e^+e^- \to \mu^+\mu^-$:

$\propto e^2$,



$\propto e^4 \propto e^2(\alpha)$

So, as we add more loops, we add nore powers of α. Therefore, Feynman diagram perturbation theory is an expansion in the fine-structure constant about $\alpha = 0$. The radius of convergence of this expansion is zero.

The argument for this is due to Freeman Dyson. For $\alpha > 0$, it costs energy for ~~an electron to emit a photon~~ a photon to split into an electron-positron pair. However, if $\alpha < 0$ (that is, electric charge is imaginary), then the system can lose energy by photons splitting to electron-positron pairs. So, the system can shed an electron-positron pair and keep losing energy. This can in principle continue ad infinitum and so the system with $\alpha < 0$ does not have a lower bound on the ground state energy.

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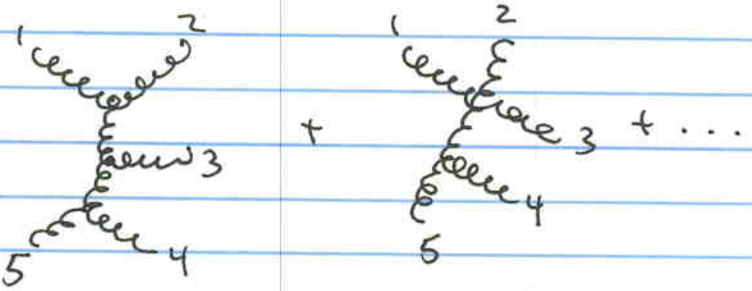
So, if Feynmandiagrams are an expansion about $\alpha = 0$, and for *any* $\alpha < 0$ there is no ground state, then the radius of convergence of the Feynman diagram perturbation theory is 0. This means that as you calculate more and more Feynman diagrams with more and more loops, the result you find does not converge. Such a series with a zero radius of convergence is called an asymptotic series.

This might seem like Feynman Diagrams are exceptionally useless. If there is no hope of convergence of the perturbation theory, then what do we do? It turns out that asymptotic series are actually exceptionally useful; often more useful than convergent series! Asymptotic series have some super crazy properties. For many asymptotic series, the result you get after a *finite* order in the perturbation theory is arbitrarily close to the exact result. This is *not* what happens with convergent series. Additionally, the precise way that asymptotic series diverge as you add more terms contains a huge amount of information for properties of the exact result. There is an effort in the theoretical physics community to understand the behavior of asymtotic series in quantum field theory, which is a program called "resurgence".

Feynman diagrams, in addition to corresponding to the expansion of an asymptotic series, also are less-than-optimal for efficiency of calculation. As we have discussed in this class, Feynman diagrams encode momentum, energy, and angular momentum at every vertex, and interactions are mediated by force-carrier bosons, that travel at most at the speed of light. Because of these properties, Feynman diagrams have a beautiful physical interpretation, but this can also come with a huge amount of baggage. To illustrate this, let's consider the calculation of the tree-level diagram for five-gluon scattering. We will label the gluons 1, 2, 3, 4, 5, and define them with a definite helicity. The Feynman diagram calculation of this process includes diagrams like:



However, to calculates this matrix element requires calculating 9 Feynman diagrams! This has something like 612 terms! This is insane and requires a computer to evaluate.

However, if you are able to simplify the result, the final answer is insanely simple. If gluons 1 + 2 have left-handed-helicity and 3, 4, and 5 have right-handed helicity, one finds the matrix element is:

$$\mathcal{M}(g_{1L}, g_{2L}, g_{3R}, g_{4R}, g_{5R}) = -ig^3 \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Here $\langle ij\rangle$ represents the spinor-helicity notation, and is just a short-hand for the spinor product:

$$\langle ij\rangle = u_R^\dagger(p_i)\, u_L(p_j)$$

This is ridiculously simple! Why are there so many Feynman diagrams, and what are Feynman diagrams hiding to make them so complicated?

So, whenever we have a result in physics that is too simple for the input given, we look for a symmetry or some deeper explanation. For the past 20 years or so, there has been a large effort in the theoretical community to develop new methods for calculation that are much more efficient than Feynman diagrams. This effort goes by the name of "Amplitudes" or "S-matrix" program. It turns out that there are deep connections between Feynman diagrams, function theory, the structure of transcendental numbers, algebraic geometry, and other fascinating mathematics.

The Future of Collider Physics

While the LHC still has about 20 years of data taking ahead of it, it is not too early to think about the next collider experiment. The last frontier collider experiment located in the United States was the Teratron, which collided protons on anti-protons from the late 1980's to 2011. Its biggest accomplishments were the discovery of the top quark in 1995 and precision measurements of the mass of the W boson. There is no plan for another collider physics experiment in the United States at any point in the future at this time.

The reason for this requires a bit of history. In the late 1980s and early 1990s a collider physics experiment was planned that would have been located in Waxahachie, Texas, and collide potons at 30 TeV; twice the center of mass energy of the LHC. There was a huge effort by experimental and theoretical physicists to understand and predict the physics from this collider. It was called the Superconducting Supercollider (SSC) and had the possibility to push the field far forward, decades ahead of the LHC. Unfortunately, it wasn't to be as with budget issues and dwindling support from congresspeople who didn't live in Texas, Congress Killed the program in 1993.

The Large Hadron Collider program was initiated in the mid 1980s, and with the termination of the SSC program, received huge support, throughout the world. After a few hiccoughs, the Large Hadron Collider has been collecting data since late 2009, exceeding most all predictions for its performance and data collection.

In the future, there are two major collider experiments proposed. The first has been in the works for about 20 years and it's called the International Linear Collider, or ILC. After the discovery of the Higgs boson at the LHC, we want to study its properties in as much detail as possible. The most controlled experimental way to do this is in an electron-positron experiment. In an e^+e^- collider, the center-of-mass collision energies is very sensitively controllable. This enables one to finely tune the collisions to produce particular final states, like $e^+e^- \to HH$, which is a process that is sensitive to the structure of the Higgs potential. There is international support for the ILC, and it is likely to be constructed in northern Japan within the next decade. The Japanese are very excited about this project; high schools have made promotional videos to support the building of the machine in their town.

The biggest future experiments that are being proposed are higher energy proton collision experiments; collisions that will have center-of-mass collisions of 100 TeV, almost 10 times the collision energy of the LHC! The efforts for these future 100 TeV machines have just started in the past few years, and the most serious efforts have been lead by China and CERN. Proposals for the location of the machine in China are being looked at now, with potential locations ranging from the end of the Great Wall, to just north of Hong Kong, to inland, near Xian, where the Terra Cotta warriors are located. The CERN site would be still located in Geneva, with the current LHC ring one of the pre-accelerators before the main ring. Both the China and CERN proposals have a main ring of 100 km, over 3 times the circumference of the LHC. The scale of these projects is exceptionally long-term, with no prediction of collisions before about 2045. However, they will be the next frontier of probing the universe at ever decreasing distances, in order to understand a bit more about how we got here.