Homework 6

Phys 411

October 4, 2018

Due: Friday, October 12

1. Consider the Schwarzschild solution of three-dimensional space-time. The metric can be written in the generic form:

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\theta^2 \,. \tag{1}$$

   Show that Einstein's equations in vacuum, $R_{\mu\nu} = 0$, require that $\alpha = \beta = 0$. That is, the most generic spherically-symmetric, three-dimensional space-time is flat. To do this, you need to calculate the Christoffel symbols, the Riemann tensor, and then the Ricci tensor.

2. Consider Einstein's equation with a cosmological constant:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \,. \tag{2}$$

   Working in four-dimensional space-time like we did in class, solve these Einstein's equations for a spherically-symmetric, static metric. You should be able to largely recycle results from class to identify the Ricci tensor, for example, but you will need to calculate the Ricci scalar and then plug it all into Einstein's equation. Make sure that in the limit that $\Lambda \to 0$, you recover the Schwarzschild solution.