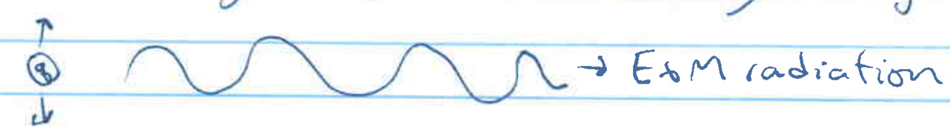
Gravitational Wave Detection

In our study of gravitational waves described by metric fluctuations, we had identified several interesting properties:

- Gravitational waves travel at the speed of light,
- Gravitational waves carry spin-2,
- Gravitational waves have two spin/polarization states.

If we could directly observe gravitational waves, and verify their properties, this would be an extremely detailed test of general relativity. Further, the observation of gravitational waves would provide a new window into astrophysical or even cosmological phenomena. The waveform of a gravitation wave, just like for electromagnetic radiation would encode information about the gravitating bodies that produce that radiation. In this lecture, we will survey this field, qualitatively describe the production and observation of gravitational radiation, and discuss the discovery a few years ago. For a more quantitative discussion, I reference you to the textbook.

The first thing we will discuss is how gravitational waves are produced. To provide context, let's go back to E&M and discuss how electromagnetic radiation is produced. E&M radiation is produced from accelerating charges. Perhaps the simplest accelerating charge we can imagine is an oscillating charge:



This oscillation represents an acceleration (i.e., non-zero second time derivative) of the dipole moment of the charge distribution. Indeed, if the displacement of the charge q is
$$\vec{d}(t) = d_0 \sin\omega t \hat{z}$$
where d_0 is the amplitude, then the dipole moment is

$$\vec{p}(t) = q\vec{d}(t) = q d_0 \sin\omega t \hat{z},$$
so its acceleration is: $\ddot{\vec{p}}(t) = -\omega^2 q d_0 \sin\omega t \hat{z}$. The total power radiated in E+M radiation is given by the Larmor formula:
$$P = \frac{\mu_0 q^2 a^2}{6\pi c},$$

where a is the magnitude of acceleration. You might have seen this in E+M last year.

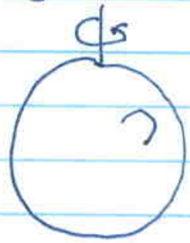
Before doing the same exercise for gravity, why is electromagnetic radiation generated by an accelerating dipole? We had argued earlier that the photon, the quanta of E+M radiation, carries spin 1. This means that under a rotation about its direction of momentum/Poynting vector, it returns to itself after a rotation of 2π. We see this from the standard picture of E+M radiation from oscillating $\vec{E}$ + $\vec{B}$ fields:

Correspondingly a dipole only returns to its original configuration after rotation by 2π:

Because the photon is spin-1, it is created by accelerating dipoles, and vice-versa.

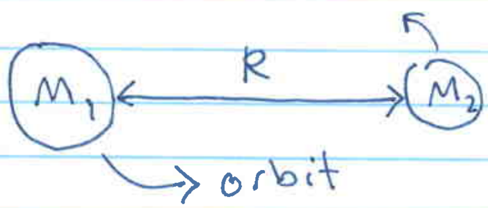
By contrast, we had argued that gravitational radiation returns to its original orientation after only a rotation of an angle π. So, the way in which gravitation radiation is produced and manifests must respect this property. In particular, gravitational radiation cannot be produced by oscillation of a mass dipole. First, such a dipole doesn't exhibit the necessary symmetry, as discussed above. Further, an oscillating dipole doesn't conserve momentum! In the E+M case, an accelerating dipole means that the center-of-charge is accelerating, like the oscillating charge. Correspondingly, an accelerating dipole in gravity means that the center-of-mass is accelerating!

Thus, gravitational radiation is produced by acceleration of the quadrupole moment of a mass distribution. Unlike the dipole, the quadrupole is the first moment that is sensitive to the shape of a system. For example, a perfect sphere set to rotate about an axis exhibits no quadrupole moment and so does not produce gravitational radiation. ~~However, if you~~ Not surprisingly, as this is described by the Kerr metric. However, if you made a bump, ever so small on the surface, that would produce a non-zero acceleration of the quadrupole moment:



We could make this bump larger and larger and further consider the limit in which it is excised.

Then, the two bodies will mutually orbit one another with a non-zero quadrupole moment.

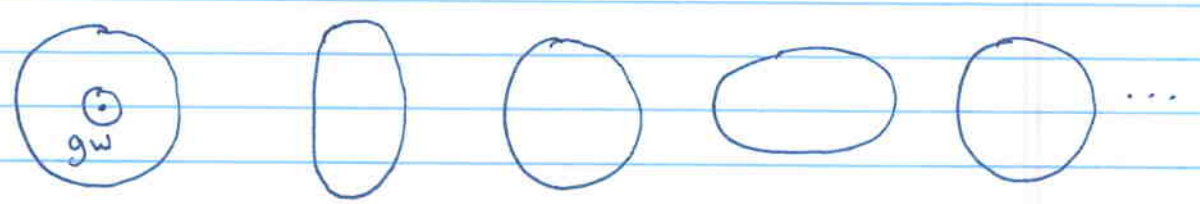


Such a binary system emits gravitational radiation. If the masses are equal $M_1 = M_2 = M$, then the "Larmor formula" for the power generated in gravitational waves by this orbiting system is:

$$P = \frac{2}{5} \frac{G_N^4 M^5}{c^5 R^5}$$

So, how do we observe gravitational waves? Essentially the opposite of how we create them. Let's go back to E+M and remind ourself there. An antenna is just a long conductor. When an E+M wave passes by, the free charges in the conductor are accelerated by the oscillating electric field. This produces an AC voltage that can be detected or transferred to an audio or visual signal. Note again the importance of the spin-1 nature of the photon. We only need a long straight conductor for our antenna because the oscillations necessarily occur in a line. Because the oscillating electric field only returns to itself after 2π rotation, this enforces oscillations in a plane. By the way, old TVs had "rabbit ear" antennas because it wasn't known what direction the electric field would be oscillating in. Multiple antennas increased the received signal, but strictly in not necessary.

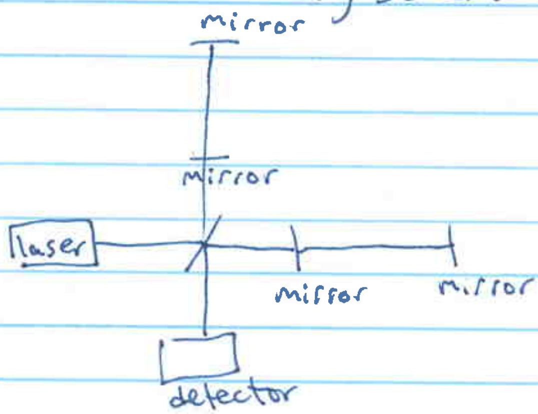
This is in contrast to gravitational waves. Just as E+M waves conserved electric charge, gravitational waves must conserve momentum. So, the effect of a gravitational wave is not to oscillate a mass, as that does not conserve momentum. Instead, gravitational waves stretch and contract space in perpendicular directions. For example, if we consider a ring through which a gravitational wave passes, it will be distorted as:



Antipodal points have equal and opposite momentum. Further, the stretching and contracting of the ring returns to itself after a rotation of π, exactly as required by a spin-2 graviton.

So, it's not sufficient to observe gravitational waves with an "antenna": one needs to observe this expansion and contraction about two perpendicular axis.

So, this is what is done. LIGO is such a gravitational wave detector. LIGO consists of two sites, one at Hanford, WA, and the other in Livingston, LA, each of which is a two-arm laser interferometer. I'll ever so briefly discuss how it works. Very schematically, LIGO is:



A laser sends coherent light to a half-silvered mirror. That light is then split to two perpendicular arms which are each about 4 km long and are the purest vacuums in the universe. The light in the arms bounces back and forth hundreds of times before being recombined in a detector. If a gravitational wave passed through, it would manifest as a phase difference between the two arms. That phase difference could be reinterpreted as the waveform of the gravitational wave. LIGO is sensitive to differences in travel distances along the two arms of order of 10^{-20} m. This is enough to detect gravitational waves.

On September 14, 2015, both LIGO sites observed identical waveforms slightly displaced in time, consistent with traveling at the speed of light. These waveforms were then used to extract information about their source: two mutually-orbiting black holes, each with mass of about 30 solar masses, which emitted gravitational waves. The peak power output in gravitational waves was approximately 10^{49} W, or about 50 times the power output of all the stars in the visible universe. The total energy output in gravitational waves was about 3 solar masses; i.e. $E \cong 3 m_{\odot} c^2$. The distance difference between the two arms of LIGO that was detected was about $1/1000^{th}$ the radius of the proton. All that power in gravitational waves had dispersed; not surprisingly because the black holes were about a billion light-years away.