Our Universe

First, evaluations!

Next, let's recall the Friedmann equations which were Einstein's equations for the maximally-spatially symmetric universe:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right].$$

The Friedmann equations are:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p)$$

Further, we had the conservation of the stress energy tensor:

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

which led to different scale factor dependence for different sources of energy density:

$$\rho \propto a^{-n}: \quad n=4 \Rightarrow \text{radiation}, \quad n=3 \Rightarrow \text{matter}$$
$$n=2 \Rightarrow \text{curvature}, \quad n=0 \Rightarrow \text{cosmo. constant}$$

In a universe like our own which has in general all of these sources of energy density, the <u>total</u> energy density is a sum of individual energy densities of each component:

$$\rho_{tot} = \sum_i \rho_i.$$ With the dependence on

scale factor identified, we can express each component as: $\rho_i = \rho_{i0} a^{-n_i}$, where ρ_{i0} is the initial energy density. Further, the derivative of the scale factor is called the Hubble parameter H:

$$H = \frac{\dot{a}}{a},$$ so we can write the Friedmann equation as:

$$\boxed{H^2 = \sum_i \frac{8\pi G_N}{3} \rho_{i0} a^{-n_i}}$$

In general, the Hubble parameter is not constant, but is at any rate measurable. Because each different component of the energy density has a different, parametric, dependence on scale factor a, in general one will dominate at any given time. So, we can approximate the Friedmann equation at one time as:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \rho_{i0} a^{-n_i},$$ for species i.

We can solve for $a(t)$ to find:

$$a(t) \propto t^{2/n_i}$$

All of the sources of dynamical energy density (matter, radiation) have values of $n > 0$; thus, for $t \to 0$, the scale factor shrinks to 0: that is, as $t \to 0$, space becomes smaller and smaller. This singularity of the spacetime is called the Big Bang.

For the cosmological constant, $n = 0$, so the solution to the Friedmann equations is:

$$a_\Lambda \propto e^{Ht},$$ or exponential expansion.

At late times, when the universe is dominated by vacuum energy, it will exhibit exponential expansion. Distances will grow enormously, and we will quickly be very alone in the universe.

This exponential expansion canbe useful, however. To explain the homogeneity and isotropy that we observe in our universe, one solution is to posit that in the early universe, shortly after the big bang there was a period of rapid expansion, called inflation. In the very early universe, because all distances would be tiny, there would be enormous fluctuations simply due to quantum mechanics. These fluctuations would have been magnified as the universe grew, unless there was a mechanism to quickly dilute them. If one postulates that the very early universe was actually vacuum energy dominated, then the universe will expand exponentially, washing out the quantum fluctuations to a large part. Those small fluctuations that remain after the period of inflation seed the components of the universe that are not homogeneous and isotropic: galaxies, stars, planets, and us.