Lab Problems

1. Write an implementation of the Metropolis algorithm for the 2D Ising Model with periodic boundary conditions. Call this function `metropolis` and have it take the same arguments as the heat bath algorithm function `heatbath` provided in the supplemental Mathematica notebook. For all of the following questions, use $J = 1$ and a spin lattice at least as large as 100×100 sites.

2. Using your `metropolis` code, visually estimate the value of T_c from the equilibrium spin configurations at various temperatures. How does this compare to the true critical temperature $T_c \simeq 2.27$?

3. Calculate the average energy per lattice site of the 2D Ising Model using your `metropolis` code. Set the temperature to be $T = 1.4$ to 3.2 in steps of 0.2. Compare what you find to Onsager's exact result:

$$\langle E \rangle = -J \coth(2\beta J) \left[1 + \frac{2}{\pi}(\sinh^2(2\beta J) - 1) \int_0^{\pi/2} \frac{d\theta}{\sqrt{\cosh^4(2\beta J) - 4\sinh^2(2\beta J)\sin\theta}} \right].$$

   The integral is another elliptic-type integral: you will need to use a numerical integrator (Simpson, Monte Carlo, etc.) to evaluate it. Plot $\langle E \rangle$ as a function of temperature from your `metropolis` code and from the exact result for the temperatures listed above. What do you notice about the plot of the energy as a function of temperature near T_c?

4. Using the spin correlation code provided in the supplemental Mathematica notebook, calculate the spin correlation for the 2D Ising model as simulated by your Metropolis algorithm code. Plot the spin correlation out to $l = 20$ for all of the temperatures that you simulated in the previous question. Using the `FindFit` function in Mathematica, fit the spin correlations at each temperature to the expected form for exponential decay:

$$C(\beta, J, l) = a + be^{-l/\xi}\,. \tag{65}$$

   Fit for the constants a and b and the correlation length ξ. What temperature that you simulated has the largest value of the correlation length ξ?